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New Goods and the Transition to a New Economy

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The U.S. went through a remarkable structural transformation between 1800 and 2000. A precipitous decline in the importance of agricultural goods in the economy was matched by the rapid ascent of a plethora of new non-agricultural goods and services. A competitive model is presented here where consumption evolves along the extensive margin. This lessens the need to rely on satiation points, subsistence levels of consumption, and the like to explain agriculture's demise. The analysis suggests that between 1800 and 2000 economic welfare grew by at least 1.5% a year, and may be as much as 10% annually, the exact number depending upon the metric preferred.

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JEL Classification: E13, O11, O41

1. Introduction

In 1800 agriculture accounted for 46% of U.S. output, while 74% of the U.S. population worked in this sector. By 2000 agriculture made up 1.4% of output. Less than 2.5% of the populace worked there. Figure 1 tells the story about the decline in agriculture.[1] What accounts for agriculture's precipitous fall? The idea here is that along with economic development many new goods are introduced. This occurs because technological progress implies that more consumption can be purchased for a unit of time spent working. As purchasing power increases, expenditure gets directed toward new products. That is, consumption moves in large measure along the extensive margin, so to speak, and not the intensive one. In a competitive world, firms will leap in to satisfy the demand for more and more new goods by consumers.

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1 The data for agriculture's share of income derives from four sources: (i) 1800–1830, Weiss (1994, Tables 1.2, 1.3 and 1.4); (ii) 1840–1900, Gallman (2000, Table 1.14); (iii) 1910–1970, *Historical Statistics of the United States: Colonial Times to 1970* (Series F 251); 1980–2000, Bureau of Economic Analysis, US Department of Commerce. The numbers from Weiss (1994) are obtained by multiplying his series on output per worker by the size of the labor force (prorated by his labor-force participation rate). The data on agriculture's share of employment comes from three sources: (i) 1800–1900, Margo (2000, Table 5.3); (ii) 1910–1960, Lebergott (1964, Tables A1 and A2); (iii) 1970–1999, U.S. Census Bureau, US Department of Commerce.

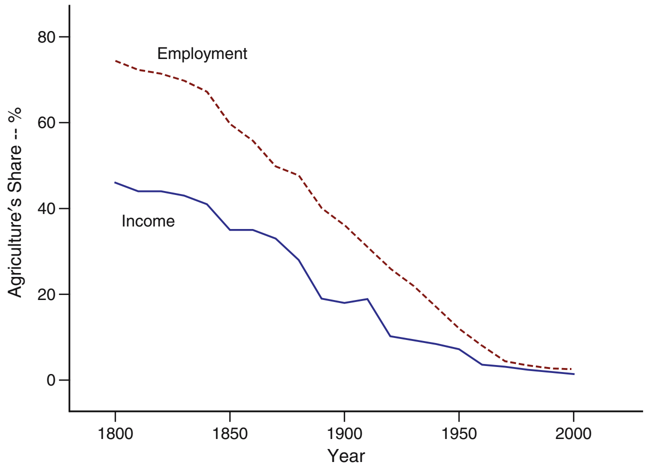

Figure 1. The Decline of Agriculture, 1800–2000.

1.1. The Analysis

Kuznets (1957) was an early researcher to report facts about agriculture, both across time and space. He documented the secular decline in agriculture's shares of output and employment for a number of countries (see his Tables 7 and 14). He also noted that agriculture declined with economic development in a cross section of countries (see his Tables 3 and 10).

Given these facts, some models have been developed that connect structural transformation with economic development. They fall into two broad, but not mutually exclusive, categories: viz taste-based models and production-based ones. Two first-rate examples of the taste-based approach are Echevarria (1997) and Laitner (2000). Laitner (2000) develops a model of the decline in agriculture and the rise in manufacturing that occurs with economic progress. His analysis relies on a satiation level for agricultural consumption. An increase in agricultural consumption provides no more utility at a certain point. At this stage individuals start consuming manufacturing goods. Echevarria's (1997) model is quite similar. In her setting the utility function for primary goods (read agricultural goods for the current purpose) is more concave than are the utility functions for manufacturing goods and services. Therefore, when poor, an individual prefers to spend most of his income on primary goods. A subsistence level for primary goods consumption would work in a similar way. Along these lines, restrictions on tastes and technology that allow for tractable solutions to growth models have been developed by Kongsamut et al. (2001). Last, Gollin et al. (2002) argue that the release of labor from agriculture, due to gains in productivity, is important for spurring on the economic development process.

A prime example of the production-based approach is Hansen and Prescott (2002). Food and manufacturing goods are perfect substitutes in utility. Agricultural goods are produced using a pre-industrial production technology that is land intensive. Manufactured goods are produced using an industrial technology that does not require land. At low levels of development it does not pay to use the industrial technology. As an economy develops the industrial technology is brought into use. Eventually, it dominates production for two reasons. First, it has a higher rate of technological progress. Second, it is unencumbered by the presence of the fixed factor, land. Instead, it uses the reproducible factor, capital, more intensively in production.

The consumption of a greater array of goods is part and parcel of economic development. This key fact is the focus of current work. The above analyses abstract from this important feature of the development process. The idea is that at higher levels of economic development it pays to bring new goods on line. This notion is contained in a classic paper by Romer (1987).[2] Both the application and formulation here are different though. Take the formulation, first. The current analysis is done within the context of a multisector model with *perfect* competition and *decreasing* returns to scale. With additively separable concave utility, the benefit from bringing a new good on line will exceed the benefit from consuming more of an old good. To limit the range of goods consumed at a point in time, it is merely assumed that there is some lumpiness in consumption.[3] This rules out the infinitesimal consumption of all goods. Romer (1987) focuses on the use of new goods in production, not consumption. He effectively limits the number of new goods that are available by assuming that each new good is produced by a monopolist, who must incur a fixed cost of production. Macroeconomists generally prefer to view the world through the competitive lens, when possible. For good reason, too; most goods are produced by more than one firm. There were hundreds of firms producing the new good, automobiles, at the turn of the last century [Klepper (2001)], just as there are hundreds of firms producing the new good, personal computers, today. In fact, the introduction of a new good is generally associated with a flood of firms into the market, followed by a period of ruthless competition whereby many firms are forced to leave (the "shake-

2 A well-known model of new goods is developed by Stokey (1988). She uses a Lancasterian characteristic model, very different from the framework developed here. Each vintage of new goods embodies all of the characteristics of previous vintages. Individuals would prefer to consume just the latest generation of goods, but they cost more. So, they consume a spectrum of goods. A nice feature of her analysis is that over time consumers drop the consumption of some older goods in favor of better newer goods. In interesting work Yorukoglu (2000) connects the development of new goods with business cycles. In his model firms must decide each period whether or not to attempt to introduce a new product. Once a product is introduced it goes through "process innovation" over time whereby it can be manufactured at lower and lower cost. His setup has interesting implications for economic fluctuations. Suppose the number of products out on the market is small relative to the size of the economy. It will be profitable for firms to attempt to introduce new products. This will lead to a burst of product innovation and a boom. Eventually, the market may become flooded with products. It then no longer pays to introduce a new product. So, product innovation stalls. Worse still, process innovation implies that the existing products can be produced at lower and lower cost. This may lead to a decline in employment. Hence, a recession ensues.

3 Yorukoglu (2000) makes this assumption too.

out'' phase). This stylized fact is documented by Gort and Klepper (1982) in a classic study of 46 product innovations. Jovanovic and MacDonald (1994) analyze this process for U.S. tire industry at the turn of the last century.

Turn now to the application. The current analysis focuses on structural change. The model developed here matches quite well the pattern of structural change observed in the U.S. data. The evidence suggests that this is inextricably linked to the introduction of new goods, as is discussed below. The framework developed lessens (or even avoids, if desired) the need to rely on satiation and subsistence points in utility. An interesting question to ask is: By how much has economic welfare increased over the last 200 years? It is easy to address this question through the eyes of the model. The answer obtained is compared with some conventional model-free measures of the rise in living standards.

1.2. Some Facts

1.2.1. New Goods

The number of goods produced has increased dramatically since the Second Industrial Revolution. The rise in the number of consumption goods is hard to document. Historically, home production accounted for a large part of consumption. For instance, 92% of baked goods were made at home in 1900.[4] This had dropped to 22% by 1965. Similarly, 98% of vegetables consumed were unprocessed, as opposed to 30% in 1970.[5] Per-capita consumption of canned fruits rose from 3.6 pounds in 1910 to 21.6 pounds in 1950.[6] In the early 1970s there were 140 vehicle models available.[7] This had risen to 260 by the late 1990s. Likewise, there were 2,000 packaged food products available in 1980 compared with about 10,800 today.[8]

1.2.2. Trademarks and the Number of Firms

Another measure of the rise in new goods is trademarks. A trademark is a symbol used by a manufacturer to distinguish his product from others. Figure 2 shows the registration of trademarks since 1870. This is a flow measure. It can be thought of as a proxy for the number of new goods introduced each year. The stock of outstanding trademarks at a point in time will be much larger. It can be

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4 See Lebergott (1976, Table 1, p. 105).
5 Ibid.
6 Ibid.
7 Federal Reserve Bank of Dallas, 1998 Annual Report, (Exhibit 3, p. 6).
8 Ibid.

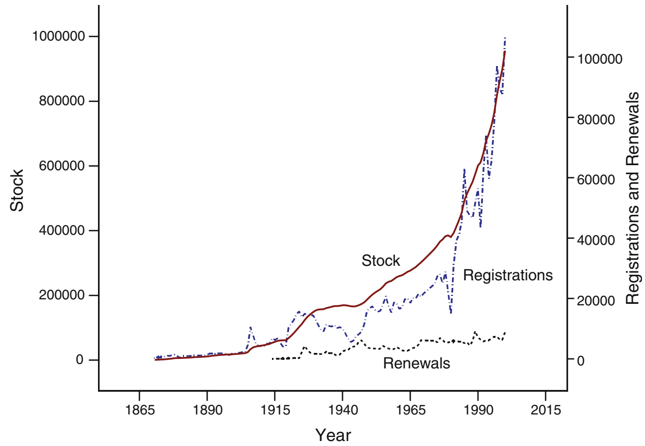

Figure 2. Estimated stock of trademarks, 1871–2000.

estimated using data on trademark registrations and renewals.[9] Likewise, one might expect that as the number of goods and services in the U.S. economy increases so will the number of firms. There is some evidence suggesting that this is the case. Figure 3 plots the number of firms per capita in the U.S. economy.[10] As can be seen, it rises.

9 For period 1891–1970 the data on registered trademarks and renewals is taken from *Historical Statistics of the United States: Colonial Times to 1970* (Series W 107 and W 108). These series are updated using data from the United States Patent and Trademark Office, US Department of Commerce, Annual Reports. The stock of trademarks is computed as follows: Let the time-t stock be denoted by t_t. The stock of trademarks is assumed to evolve in line with

$$t_{t+1} = \delta t_t + [i_t + r_t],$$

where i_t represents new registrations at time t, r_t is renewals, and δ is the depreciation factor on trademarks. Trademarks need to be renewed roughly every 20 years. Most of them are not. Now, represent the mean of $r_t/(r_{t\text{-}20}+i_{t\text{-}20})$ by $\overline{r_t/(r_{t-20}+i_{t-20})}$. This measures the survival rate on trademarks. The depreciation factor on trademarks is then taken to be given by

$$\delta = [\overline{r_t/(r_{t-20}+i_{t-20})}]^{1/20}.$$

10 This evidence is based on income tax receipts: *Historical Statistics of the United States: Colonial Times to 1970* (Series V 1) and the corresponding updated data taken from the Internal Revenue Service, U.S. Department of the Treasury. This data encompasses virtually all business in the U.S. and includes corporations, partnerships, and non-farm sole proprietorships. Evidence based on data taken from Dun & Bradshaw, Inc. shows that the number of firms per capita has remained constant—*Historical Statistics of the United States: Colonial Times to 1970* (Series V 20). The latter series is probably the least preferable and is biased toward large firms. It is based on financial market dealings and excludes many types of business—those engaged in amusements, farming, finance, insurance, one-man services, professions, and real estate. The series for the number of firms is deflated by size of the population as recorded in the *Statistical Abstract of the United States* (2001, Table 1).

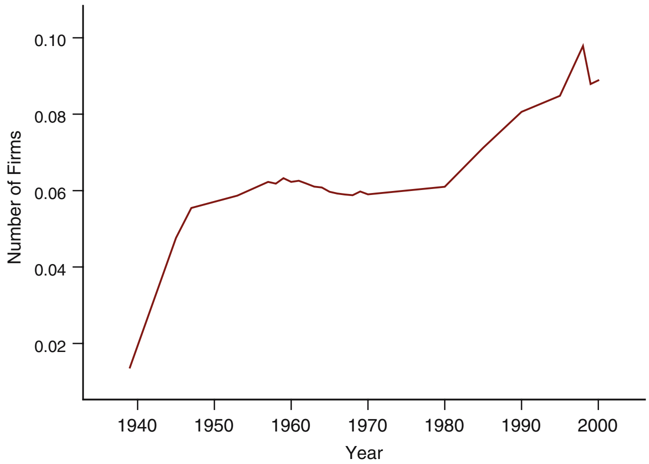

Figure 3. Number of firms per capita, 1939–2000.

1.2.3. Consumer Expenditure Patterns

Figure 4 traces some major categories of Personal Consumption Expenditure taken from the National Income and Product Accounts.[11] At the turn of the last century spending on food accounted for 44% of the household budget. Today it is 15%. The decline in food's share of total expenditure was matched by a rise in spending in other categories, such as medicine, personal business, recreation and transportation. The only category showing a secular decline similar to food is clothing, accessories and services (which is not plotted separately, but is included in the "other" category). Until recently most expenditure categories were small relative to food. Spending on medical care, which shows a rapid increase, now exceeds spending on food. Clearly the rise in medical spending was associated with the development of new goods. Figure 5 makes this point clear with a chronology of medical innovations.[12] Likewise, Figure 6 plots expenditure on

11 Source: National Income and Product Accounts, Personal Consumption Expenditure by Type of Product, Table 2.6, Bureau of Economic Analysis, US Department of Commerce. The numbers for 1900–1929 are taken from Lebergott (1996, Table A1).

12 The sources for the data used to calculate the shares of personal consumption expenditure in Figures 5–7 are the same as in the previous figures. The timelines were constructed from sources on the internet. Since these web sites are too often transient in nature, copies of the web pages (which are too numerous to list) are available from the authors. The dates in Figure 5 are: 1901, Electrocardiograph; 1916, Plastic Surgery; 1920, Radiotherapy; 1922, Insulin; 1927, Iron Lung and Contact Lens; 1928, Fibre-optic Imaging; 1932, Defibrillator; 1933, Gas and Air Apparatus; 1936, Prontosil and Ice-Pick Lobotomy; 1940, Hormones; 1941, Penicillin; 1942, Estrogen Pill; 1945, Artificial Kidney and Flu Vaccine; 1949, Cortisone; 1953, PET Scanner; 1954, Kidney Transplant, Polio Vaccine and Nystatin;

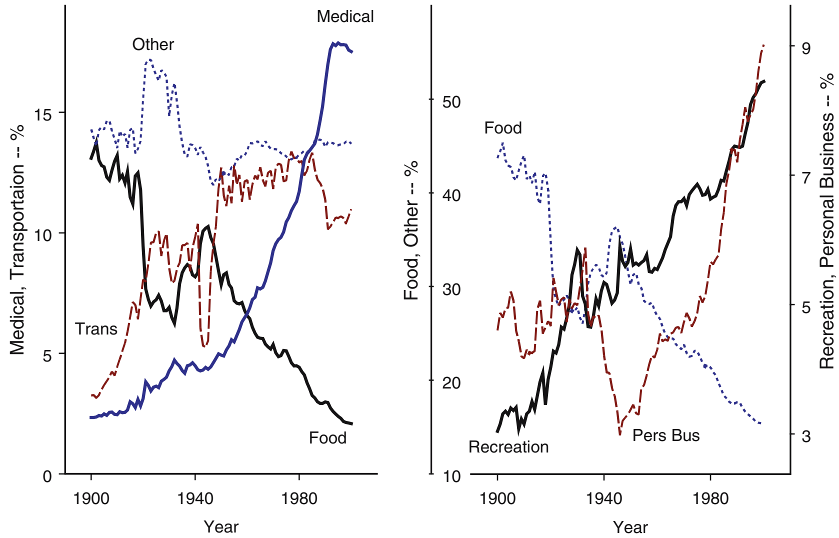

Figure 4. Expenditure shares by major catergories, 1900–2000: Purchased Food; Medical Care; Personal Business; Recreation; Transportation; Other (Clothing, Accessories and Services; Education; Household Operation; Housing; Personal Care; and Religion and Welfare).

electricity, a component of the near stationary household operations category (which again is not graphed separately in Figure 4, but is included in the other category).[13] While electricity is a relatively small fraction of the household budget, it shows a strong upward trend over the last 100 years, linked with the development of many new electrical goods. Last, over the last century total

1955, Ultrasound, Tetracycline and The Pill; 1956, Plastic Contact Lenses; 1957, Blood-Heat Exchanger and Anti-Depressants; 1958; Human Growth Hormone and Endoscopy; 1960, Laser and Implanted Pacemaker; 1962, Joint Replacement Surgery; 1963 Measles Vaccine and Liver Transplant; 1964, Coronary Artery By-Pass; 1965, Balloon Catheter; 1967, Heart Transplant; 1968, CAT Scanner; 1969, In Vitro Fertilization; 1971, MRI; 1973, Computerized Tomography; 1974, Ibuprofen; 1976, Glucometer; 1978, Test-Tube Baby; 1980, Cylcosporine; 1982, Artificial Heart and Hepatitis B Vaccine; 1985, Keyhole Surgery; 1986, Synthetic Skin and Synthetic HGH; 1988, MMR Vaccine, Laser Eye Surgery; 1990, Day-Case Surgery; 1996, Protease Inhibitor Cocktails.

13 The dates in Figure 6 are: 1900, Stove; 1903, Iron; 1908, Coffeemaker, Vacuum Cleaner and Washing Machine; 1916, Refrigerator and Electric Heating; 1917, Standardized Plugs and Portable Drill; 1919, Pop-up Toaster and Superheterodyne Radio; 1921, Electric Blankets; 1925, Record Player; 1927, Garbage Disposer; 1928, Handsaw; 1930, Kettle and Mixmaster; 1931, Razor; 1935, Clothes Dryer; 1937, Blender, Hand-Held Vacuum; 1946, TV and Central Air; 1947, Tape Recorder; 1951, Hair Dryer; 1955, Deepfreezer; 1959, Dishwasher; 1965, Microwave; 1971, Food Processor; 1975, VCR ; 1979, Video Disc; 1981, IBM PC; 1984, CD Player; 1995, DVD.

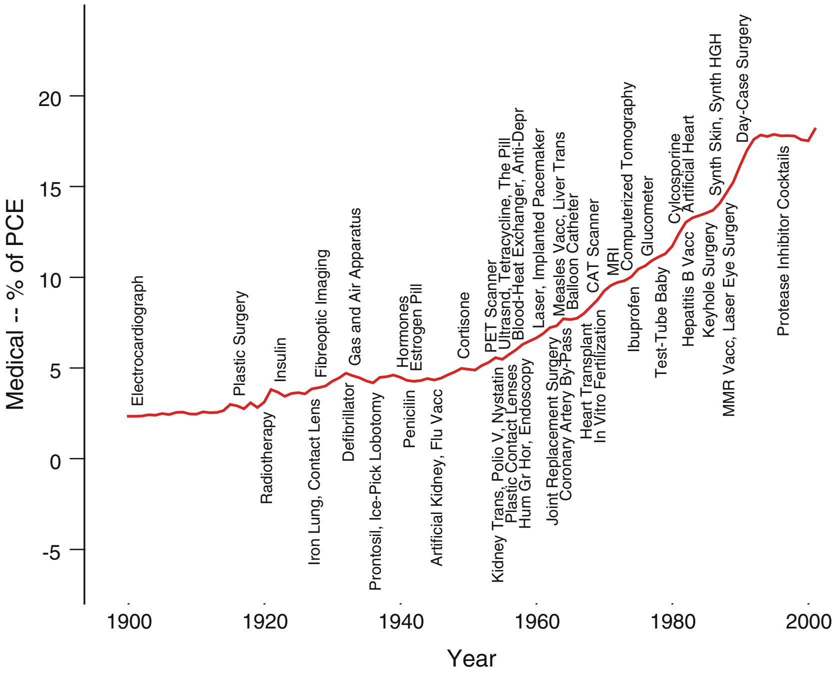

Figure 5. Medicine, 1900–2000.

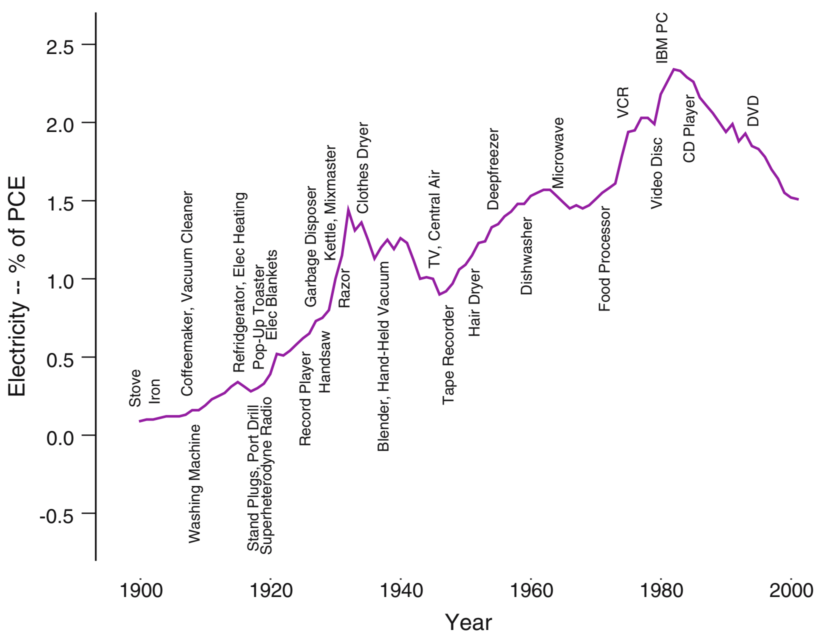

Figure 6. Electricity, 1900–2000.

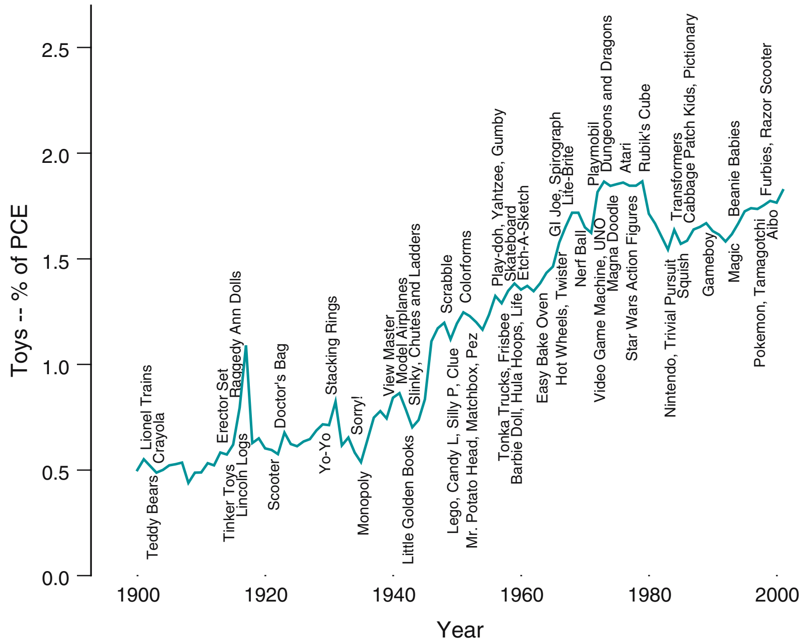

Figure 7. Toys, 1900–2000.

recreation has increased its share in the household budget. Figure 7 shows spending on toys, a component of this category.[14]

2. The Model

2.1. Tastes and Technology

The world is described by a three-sector overlapping-generations model. An individual lives for two periods. The first sector in the economy produces agricultural goods. The second manufactures a generic good, and the last sector produces new goods.

14 The dates are: 1901, Lionel Trains; 1902, Teddy Bears; 1903, Crayola; 1913, Erector Set; 1914, Tinker Toys; 1915, Raggedy Ann Dolls; 1916, Lincoln Logs; 1921, Scooter; 1922, Doctor's Bag; 1929, Yo-Yo; 1930, Stacking Rings; 1934, Sorry!; 1935, Monopoly; 1939, View Master; 1941, Model Airplanes; 1942, Little Golden Books; 1943, Slinky, and Chutes and Ladders; 1948, Scrabble; 1949, Lego, Candy Land, Silly Putty and Clue; 1951, Colorforms; 1952, Mr. Potato Head, Matchbox and Pez; 1956, Play-Doh, Yahtzee and Gumby; 1957, Tonka Trucks and Frisbee; 1958, Skateboard; 1959, Barbie Doll, Hula Hoops and Life; 1960, Etch-a-Sketch; 1963, Easy Bake Oven; 1965, GI Joe and Spirograph; 1966, Hot Wheels and Twister; 1967, Lite-Brite; 1969, Nerf Ball; 1971, Playmobil; 1972, Video Game Machine and UNO; 1973, Dungeons and Dragons; 1974, Magna Doodle; 1976, Atari; 1977, Star Wars Action Figures; 1979, Rubik's Cube; 1983, Nintendo and Trivial Pursuit; 1984, Transformers; 1985, Squish; 1986, Cabbage Patch Kids and Pictionary; 1989, Gameboy; 1993, Beanie Babies and Magic; 1997, Pokemon and Tamagotchi; 1998, Furbies and Razor Scooter; 1999, Aibo.

2.1.1. Tastes

Represent the momentary utility function for a person by

$$\alpha \ln(a) + \psi \ln(c) + \sigma \int_{i=0}^{N} \ln(\max(s_i, \underline{s}))di,$$
$$\text{with } 0 < \alpha, \psi, \sigma < 1 \text{ and } \alpha + \psi + \sigma = 1. \quad (1)$$

Here a is the quantity consumed of agricultural goods. Each person also consumes a generic manufacturing good, c. The quantity consumed of new good i is denoted by s_i. The term $\underline{s}$ represents a lower bound on new goods consumption. For whatever reason, in the real world there does seem to be some lumpiness in the consumption of goods. This would arise endogenously if there are fixed costs associated with purchasing or consuming a good (or for that matter producing each unit). Without this assumption an individual would unrealistically desire to consume some amount of all goods, so long as prices are finite, albeit perhaps in infinitesimal quantities. With this assumption an individual will want to consume a determinate number of new goods, given a particular set of prices. Additionally, this assumption permits utility to be defined when some goods are not consumed.[15, 16] The variable N represents the upper bound on the number of new goods that can *ever* be produced. Utility would be unbounded without such a limit on the number of new goods. Hence, tastes would have to be modified to allow for a situation where new goods are perpetually coming

15 Any properly specified new-goods model must define utility when some new goods are not consumed. To illustrate the issue, consider a utility function over new goods of the form $\sigma N \ln[(1/N) \int_{i=0}^{N} s_i^{\rho} di]^{1/\rho}$, for $\rho \leq 1$. This utility function is often adopted in Romer-style new-goods models. Observe that when $\rho = 0$ one gets a logarithmic utility function of the form employed in (1), ignoring the presence of the lower bound; i.e., when $\max(s_i, \underline{s})$ is replaced by s_i. While this setup may appear to be more general than the one used here, note that for the purposes at hand, this utility function will not be suitable for use when $\rho \leq 0$ – when degree of curvature is greater than or equal to the ln case. In this situation utility is not well defined when $s_i = 0$ for some i. This is typically finessed by ignoring the zero terms in the utility function. That is, by defining the utility function to be $\sigma N \ln[(1/N) \int_{\mathcal{N}} s_i^{\rho} di]^{1/\rho}$, for $\rho \leq 1$, where $\mathcal{N} = \{i : s_i > 0\}$. In the logarithmic case this amounts to saying that zero consumption of good i yields zero utility. Now, if this is strictly true then no one would consume less than one unit of i, since this yields negative utility; i.e., $\ln(s_i) < 0$ when $s_i < 1$. Therefore, this implicitly sets a lower bound on consumption of $\underline{s} = 1$. Hence, when this assumption is explicitly taken into account the analysis proceeds along the lines developed here. Even when $0 < \rho < 1$, $lim_{s_i \to 0} d \ln[(1/N) \int_{i=0}^{N} s_i^{\rho} di]^{1/\rho} / ds_i = \infty$. This has the unrealistic feature that an individual will consume all goods so long as prices are finite, albeit perhaps some in infinitesimal quantities.

16 One might be tempted to use a utility function of the form $\ln(s + \underline{s})$, such as adopted by Kongsamut et al. (2001). The reasoning might be that s can go to zero, since marginal utility is bounded above by $1/\underline{s}$. In fact with this formulation *s will be negative* at low levels of development, unless a Kuhn–Tucker condition that is usually *ignored* is imposed. Additionally, even with a constraint of the form $s \geq 0$, in a symmetric equilibrium where all new goods sell at the same price a person would desire to consume the whole spectrum of available goods, even if the consumption of some of them was in infinitesimal amounts.

on line. Last, the phrase "new good" is not perfect since yesteryear's new good is today's old one. Despite this the analysis will proceed using this nomenclature.

2.1.2. *Sources and Uses of Income*

All individuals supply one unit of labor. They work only when young and earn the wage w. An individual can use his income for consumption or savings. Savings is done using bonds, b, which pay gross interest at rate r. These bonds are backed by capital. Agricultural goods and new goods can be purchased at the prices p_a and p_i.

2.1.3. *Production*

The output of agricultural goods, y_a, is governed by a standard Cobb–Douglas production function,

$$y_a = z_a k_a^{\lambda} l_a^{1-\lambda},$$

where k_a and l_a are the quantities of capital and labor hired in agriculture. Likewise, y_c units of the generic manufacturing good can be produced using k_c units of capital services and l_c units of labor according to

$$y_c = z_c k_c^{\omega} l_c^{1-\omega}.$$

Output from this sector is used for both consumption and capital accumulation.

Finally, a type-i new good is produced in line with

$$y_i = z_i k_i^{\kappa} l_i^{\tau}, \text{ with } \kappa + \tau < 1. \tag{2}$$

There is a fixed cost, ϕ, associated with the production of each new good i. This cost is in terms of labor. It could just as easily been expressed in terms of the generic good, although then its bite would decline over time as the economy becomes wealthier. The idea is that with this fixed cost there will be a determinate number of firms in equilibrium. This feature is needed to match, at least in a meaningful way, the observation that the number of firms per capita in the U.S. economy has grown over time—Figure 3. To cover the fixed cost, firms must earn profits after meeting their variable costs. To this end, assume that there are decreasing returns to scale in production (i.e., $\kappa + \tau < 1$). There is free entry into all production activity. The number of specialized firms will be determined by a zero-profit condition. Denote the number of firms that produce the new good i by n_i. Assume that total factor productivity is common across all types of new goods so that $z_j = z_i$ for all $j \in [0,N]$.

2.1.4. *Capital Accumulation*

At a point in time the aggregate stock of capital will be represented by $\mathbf{k}$. The law of motion for capital is described by

$$\mathbf{k}' = \delta\mathbf{k} + \mathbf{i},$$

where δ is the factor of depreciation and $\mathbf{i}$ represents gross investment (in terms of the generic manufacturing good). There is free mobility of capital across sectors.

2.1.5. Technological Progress

Technological progress will be captured by growth in z_a, z_c, and z_i. As z_i rises it becomes easier to recover the fixed costs associated with producing new goods. As z_a and z_c also grow so does consumer income, and hence the demand for a greater number of new goods. Therefore, the number of new goods produced will increase over time. This leads to a natural decline in agriculture's share of the economy. Presuppose that z_a, z_c, and z_i rise over time to some finite upper bounds and remain there forever after. Under this assumption the economy will eventually converge to some steady state.

2.2. A Young Worker's Optimization Problem

How will a young worker choose his consumption plan? Given the form of preferences (1), it is clear that if a young worker consumes new good i then he will set $s_i \geq \underline{s}$. Without loss of generality, order the new goods from the lowest to the highest price and assume that a young worker chooses to consume the first I new goods when young, and the first $I^{o\prime}$ when old. A young worker's optimization problem can then be written as

$$\max_{a, a^{o\prime}, c, c^{o\prime}, s_i \geq \underline{s}, s_i^{o\prime} \geq \underline{s}, I, I^{o\prime}} \Big\{ \alpha \ln(a) + \psi \ln(c) + \beta\alpha \ln(a^{o\prime}) + \beta\psi \ln(c^{o\prime}) + \sigma \int_{i=0}^{I} \ln(s_i) di + \beta\sigma \int_{i=0}^{I^{o\prime}} \ln(s_i^{o\prime}) di + \sigma(N - I) \ln(\underline{s}) + \beta\sigma(N - I^{o\prime}) \ln(\underline{s}) \Big\}, \tag{3}$$

subject to

$$c + p_a a + \frac{c^{o\prime}}{r'} + \frac{p'_a a^{o\prime}}{r'} + \int_{i=0}^{I} p_i s_i di + \int_{i=0}^{I^{o\prime}} \frac{p'_i}{r'} s_i^{o\prime} di = w. \tag{4}$$

Here the superscript "o" denotes an allocation when old while the "$\prime$" signifies that a variable's value next period is being considered. This problem is more or less standard with one twist: the determination of the number of new goods to consume.

2.2.1. The Consumption of Each Good

Given logarithmic structure for preferences, it is easy to solve for the quantity consumed of each good. The solution for c is given by

$$c = \frac{\psi}{\alpha + \psi + \beta\alpha + \beta\psi + \sigma I + \beta\sigma I^{o\prime}} w. \tag{5}$$

Likewise, the solutions for s_i and $s_i^{o\prime}$ read

$$p_i s_i = \frac{\sigma}{\alpha + \psi + \beta\alpha + \beta\psi + \sigma I + \beta\sigma I^{o\prime}} w, \tag{6}$$

and

$$\frac{p_i' s_i^{o\prime}}{r'} = \frac{\beta\sigma}{\alpha + \psi + \beta\alpha + \beta\psi + \sigma I + \beta\sigma I^{o\prime}} w, \tag{7}$$

at least when $s_i > \underline{s}$ and $s_i^{o\prime} > \underline{s}$. In the equilibrium being developed all new goods will sell at the same price, p_i, so that $p_j = p_i$ for all produced j. Hence, $s_j = s_i$ for all consumed j; likewise, $s_j^{o\prime} = s_i^{o\prime}$ for all consumed j.

2.2.2. *The Number of New Goods*

The first-order conditions for the number of new goods consumed each period, I and $I^{o\prime}$, are given by

$$\sigma[\ln(s_I) - \ln(\underline{s})] \leq \frac{\psi}{c} p_I s_I \text{ (with equality if } I > 0), \tag{8}$$

and

$$\beta\sigma[\ln(s_{I^{o\prime}}^{o\prime}) - \ln(\underline{s})] \leq \frac{\psi}{c} p_{I^{o\prime}}' \frac{s_{I^{o\prime}}^{o\prime}}{r'} \text{ (with equality if } I^{o\prime} > 0). \tag{9}$$

Take expression (8). The value of an extra good is $\sigma[\ln(s_I) - \ln(\underline{s})]$, the left hand side. This good costs $p_I s_I$. To convert this cost into utility terms multiply by the marginal utility of first-period consumption or ψ/c to get $\psi p_I s_I/c$, the right hand side. Using (5), (6) and (7) it conveniently follows that[17]

$$\begin{aligned} s_I &= e\underline{s}, \text{when } I > 0, \\ s_{I^{o\prime}}^{o\prime} &= e\underline{s}, \text{when } I^{o\prime} > 0. \end{aligned} \tag{10}$$

17 Observe that as the lower bound $\underline{s}$ approaches zero the quantity of new good I consumed, s_I, becomes infinitesimal. That is, as $\underline{s}$ falls the individual would like to consume more new goods by consuming less of each new good. Without a lower bound on consumption, $\underline{s}$, the individual would like to consume the whole spectrum of new goods, albeit in infinitesimal quantities as N becomes large. This is true in a Romer-style model, too. In the current setting with perfect competition, as $\underline{s}$ declines the number of firms producing each new good will decline. In Romer (1987) this is precluded by the monopoly assumption that restricts the number of firms producing each good to be one. This limits the total number of goods that can be produced.

Now, when will (8) and (9) hold with strict equality? It is easy to deduce that both equations can hold tightly only when $p_I = p'_{I^{o\prime}}/(r'\beta)$. If $p_I < p'_{I^{o\prime}}/(r'\beta)$ then only (8) can hold with equality. In this situation it is optimal to consume new goods just when young so that $I^{o\prime} = 0$. To summarize:

$$\begin{array}{ll} I \geq 0 \text{ and } I^{o\prime} = 0, & \text{if } p_I < p'_{I^{o\prime}}/(r'\beta), \\ I \geq 0 \text{ and } I^{o\prime} \geq 0, & \text{if } p_I = p'_{I^{o\prime}}/(r'\beta), \\ I = 0 \text{ and } I^{o\prime} \geq 0, & \text{if } p_I > p'_{I^{o\prime}}/(r'\beta). \end{array} \tag{11}$$

In the subsequent analysis only the first two cases transpire. These two cases will be referred to as Zone 1 and Zone 2.

2.2.3. Discussion

Some intuition for the solution to the consumer's problem (3) can be gleaned from Figure 8. For expositional purposes, assume that the economy is in Zone 1 and let all new goods sell at the same price p_I—again, an assumption that will be met in the equilibrium under study. Now, consider the decision to consume the marginal new good, I. How much of new good I should the agent purchase: $s_I = 0$, which amounts to not consuming, or some quantity $s_I \geq \underline{s}$? The utility that an agent derives from consuming more of new good I is shown on the diagram. If the consumer does not buy I he realizes the utility level $\ln(\underline{s})$, indicated by the rectangle. Alternatively, if he buys the good then he will purchase more than $\underline{s}$ and experience the utility level $\ln(s_I)$. Utility then rises in the fashion shown by the concave utility function UU'.

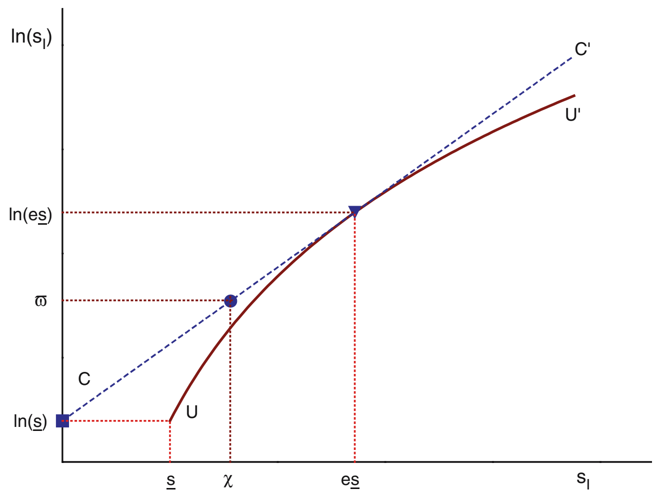

Figure 8. The determination of s_I.

The cost of consuming new good I is shown by the straight line CC'. First, by consuming I the agent loses the automatic utility level $\ln(\underline{s})$, so to speak, associated with not consuming it—cf. (3). Second, by buying more of new good I the agent diverts expenditure away from consuming more of the other new goods. These goods cost the same as I and have a marginal utility of $1/s_i = 1/(e\underline{s})$, the slope of the line CC'. The individual will pick the consumption quantity, s_I, that equates marginal benefit and marginal cost. This will be the level associated with the point of tangency between the two lines (as shown by the inverted triangle). Here, $1/s_I = 1/(e\underline{s})$ so that $s_I = e\underline{s}$.[18]

There are three types of goods in tastes: agricultural goods, generic manufacturing goods, and new goods. Over time there have been some new products in the food category, as was discussed. So, the idea here is that movement along the extensive margin has been more important for nonagricultural goods than for agricultural ones. Thus, all consumption in agricultural goods has been modeled along the intensive margin. One could model agricultural consumption along the extensive margin. Suppose there are two types of new goods, viz. agricultural and non-agricultural ones. Consumers would then have to decide on how many new goods to consume in each category. Alternatively, one could simply assume that there is only one category of new goods and that the first A new goods contained in $[0,N]$ are agricultural ones. Agricultural consumption would then just grow to $Ae\underline{s}$ and stop, other things equal. This would have the flavor of Gollin et al. (2002).[19] The generic good has been added to the framework merely to handle

18 The solution to consumer's problem has a similarity to employment lotteries, à la Rogerson (1988). There is a nonconvex region in preferences, as Figure 8 clearly shows. The individual convexifies this by moving along extensive margin. He consumes some new goods, and not others. In the equilibrium under study, for each new good $i \in [0,N]$ the individual can be thought of choosing the quantity s_i from the two-point set $\{0, e\underline{s}\}$. He randomly picks some new goods on the $[0,N]$-spectrum to maximize his utility. Let him choose to consume the fraction I/N of new goods when young, and the fraction $I^{o\prime}/N$ when old. His decision problem can be formulated as

$$\max_{a, a^{o\prime}, c, c^{o\prime}, I/N, I^{o\prime}/N} \{\alpha \ln(a) + \psi \ln(c) + \beta\alpha \ln(a^{o\prime}) + \beta\psi \ln(c^{o\prime}) + \sigma N[\frac{I}{N}\ln(e\underline{s}) + (1-\frac{I}{N})\ln(\underline{s})] + \beta\sigma N[\frac{I^{o\prime}}{N}\ln(e\underline{s}) + (1-\frac{I^{o\prime}}{N})\ln(\underline{s})]\},$$

subject to

$$c + p_a a + \frac{c^{o\prime}}{r'} + \frac{p_a' a^{o\prime}}{r'} + \frac{I}{N} N p_i (e\underline{s}) + \frac{I^{o\prime}}{N} N \frac{p_i'}{r'}(e\underline{s}) = w.$$

The new goods part of the solution to this problem is represented by the circle in Figure 8. Here, the individual can be thought of as realizing the level of utility $\varpi = [(I/N)\ln(e\underline{s}) + (1 - I/N)\ln(\underline{s})]$ that derives from consuming the convex combination of new goods $\chi = (I/N) \times (e\underline{s}) + (1 - I/N) \times 0$.

19 In fact, the current analysis has a bit of this flavor too. Using the first-order conditions for a and c, in conjunction with the fact that $s_i = e\underline{s}$, leads to

$$a = \frac{\alpha}{\sigma}\frac{p_i}{p_a}e\underline{s},$$

investment in capital in an easy way. Without this, one would have to specify how investment goods are produced from the myriad of new goods.[20] Later on when matching up the implications of model with observations from the data, the generic manufacturing good and new goods will be lumped together into a single category, non-agricultural goods.

2.3. *Firms' Problems*

First consider the firm in the generic manufacturing sector. Its problem is

$$\pi_c = \max_{l_c, k_c}[z_c k_c^{\omega} l_c^{1-\omega} - w l_c - (r-\delta)k_c]. \tag{12}$$

Next, the problem facing a firm in the agricultural sector can be written as

$$\pi_a = \max_{l_a, k_a}[p_a z_a k_a^{\lambda} l_a^{1-\lambda} - w l_a - (r-\delta)k_a]. \tag{13}$$

Perfect competition implies that factors will be paid their marginal products. Euler's theorem then guarantees that there will be zero profits so that $\pi_c = \pi_a = 0$. From the solution to problem (12), it is easy to deduce that the wage rate can be expressed as a function of the return on capital and the level of TFP in the generic manufacturing sector. The solution to problem (13) then implies that the price of agricultural goods can be expressed as a function of the return on capital, and the levels of TFP in the agricultural and generic manufacturing goods sector. Hence, write $w = W(r-\delta; z_c)$ and $p_a = P_a(r-\delta; z_a, z_c)$.[21]

Finally, turn to the production of new goods. The problem here is

$$\pi_i = \max_{l_i, k_i}[p_i z_i k_i^{\kappa} l_i^{\iota} - w l_i - w\phi - (r-\delta)k_i]. \tag{14}$$

Now, free entry into the production of new goods guarantees that profits will be zero. Therefore,

$$\pi_i = 0. \tag{15}$$

The zero-profit condition in conjunction with the solution to the firm's problem allows for the price of new goods to be expressed as a function of the return on

at least when new goods are consumed by the young (or when $s_i > 0$). Therefore, if prices are fixed then so is agricultural consumption, since one would always prefer to consume an extra new good than more food. Food consumption could rise or fall over time depending on what happens to the relative price of food in terms of new goods, p_i/p_a. Interestingly, Lebergott (1993, p. 77) argues the food consumption has fallen by 350 pounds a year—but life is probably more sedentary too.

20 A simple assumption might be to assume that each new good can be transformed in a one-to-one manner into capital.

21 The interested reader is referred to equations (A.1) and (A.8) in the Appendix.

capital, the real wage rate, and the level of TFP. One can therefore write $p_i = P_i(r-\delta; z_c, z_i)$.[22] Since $z_j = z_i$ for all produced j, and P_i is not a function of i, it transpires that $p_j = p_i$. Note that there is really just one price to worry about, r.

2.4. Market-Clearing Conditions

The markets for goods and factors must clear each period. Take the goods markets first. The market-clearing condition for the generic manufacturing good is

$$c + c^o + \mathbf{k}' - \delta\mathbf{k} = y_c, \tag{16}$$

while the one for agriculture appears as

$$a + a^o = y_a. \tag{17}$$

The market for each new good requires that

$$\mu_i s_i + \mu_i^o s_i^o = n_i y_i, \tag{18}$$

where μ_i denotes the fraction of a generation that will consume good i. Note that in order to have a symmetric equilibrium, the demand must be same for each new good produced. Now, the total number of new goods produced in a period is given by max (I,I^o). The young generation consumes the fraction $0 \le I/\max(I,I^o) \le 1$ of these goods. If each young worker randomly picks his I goods from the $\max(I,I^o)$ being offered then $\mu_i = I/\max(I,I^o)$.[23] Similarly, $\mu_i^o = I^o/\max(I,I^o)$. Now, suppose that $p_i < p_i'/(r'\beta)$; i.e., that the economy is in Zone 1. Then, $\mu_i = 1$ and $\mu_i^{o\prime} = 0$. Alternatively, if $p_i = p_i'/(r'\beta)$ it may transpire that $0 < \mu_i, \mu_i^{o\prime} < 1$.

The factor market conditions appear as

$$k_a + k_c + \max(I, I^o) n_i k_i = \mathbf{k}, \tag{19}$$

and

$$l_a + l_c + \max(I, I^o) n_i l_i + \max(I, I^o) n_i \phi = 1. \tag{20}$$

Definition *A competitive equilibrium is a set of time paths for consumption,* $\{a_t, a_t^o, c_t, c_t^o, s_{i,t}, s_{i,t}^o, I_t, I_t^o\}_{t=0}^{\infty}$, *labor and capital inputs,* $\{l_{a,t}, l_{c,t}, l_{i,t}\}_{t=0}^{\infty}$ *and* $\{k_{a,t}, k_{c,t}, k_{i,t}\}_{t=0}^{\infty}$, *the number of firms producing new goods,* $\{n_{i,t}\}_{t=0}^{\infty}$, *and interest rates,* $\{r_t\}_{t=0}^{\infty}$ *such that for an initial capital stock,* $\mathbf{k}_0$, *a time path for total factor productivities,* $\{z_{a,t}, z_{c,t}, z_{i,t}\}_{t=0}^{\infty}$, *and the pricing functions,* $W(\cdot)$, $P_a(\cdot)$, $P_i(\cdot)$:

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22 For more detail, see equation (A.2) in the Appendix.

23 In other words think about the index i in (3) as representing each young worker's personal numbering scheme over the new goods available in the first and second periods of his life. That is, out of the max (I,I^o) new goods available in the first period of his life he can choose to order them as he wishes on the interval $[0,\max(I,I^o)]$. The same is true for the second period.

1. *The consumption allocations,* $\{a_t, a^o_{t+1}, c_t, c^o_{t+1}, s_{i,t}, s^o_{i,t+1}, I_t, I^o_{t+1}\}_{t=0}^{\infty}$, *solve the consumer's problem (3), given the path for prices* $\{W(r_t-\delta;\, z_{c,t}),\, P_a(r_t-\delta;\, z_{a,t}, z_{c,t}),\, P_i(r_t-\delta;\, z_{c,t}, z_{i,t}),\, r_t\}_{t=0}^{\infty}$.

2. *The factor allocations,* $\{l_{a,t}, l_{c,t}, l_{i,t}\}_{t=0}^{\infty}$ *and* $\{k_{a,t}, k_{c,t}, k_{i,t}\}_{t=0}^{\infty}$, *solve the firms' problems (12) to (14), given the path for prices* $\{W(r_t-\delta;\, z_{c,t}),\, P_a(r_t-\delta;\, z_{a,t}, z_{c,t}),\, P_i(r_t-\delta;\, z_{c,t}, z_{i,t}),\, r_t\}_{t=0}^{\infty}$.

3. *There are zero profits in the new goods markets as dictated by (15).*

4. *All goods and factor markets clear so that equations (16)–(20) hold.*

3. Calibration

Can the above framework explain the rise of manufacturing and the decline of agriculture that occurred over the last two hundred years? In order to address this question, two things must be done prior to simulating the model. First, the engine of change in the model is technological progress. Thus, series for total factor productivity in agriculture and non-agriculture must be inputted into the simulation. Hence, to answer the question, some discussion on the extent of technological progress in agriculture and manufacturing over the 1800–2000 period of interest is in order. Second, values must be picked for model's various parameters. This will be done by benchmarking an initial and final steady state for the model to the U.S. data for the years 1800 and 2000.

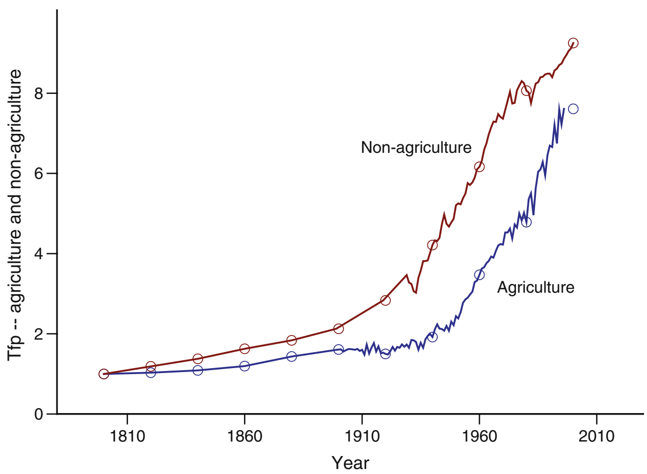

Figure 9. Total factor productivity in agriculture and non-agriculture, 1800–2000.

3.1. *Technological Progress in Agriculture and Non-Agriculture*

Take agriculture first. Total factor productivity (TFP) grew at 0.48% per year between 1800 and 1900.[24] Its annual growth rate fell to 0.26% in the interval 1900–1929 and then rose to 2.24% over the 1929–1960 period.[25] Between 1960 and 1996 it grew at an annual rate of 2.18%.[26] Hence, by chaining these estimates together, it is easy to calculate that TFP increased by a factor of 7.61 between 1800 and 1996. TFP in the non-agricultural sector—labeled manufacturing—rose at a faster clip. It grew at an annual rate of 0.75% over the period 1800–1900.[27] Its growth rate then picked up to 1.63% across 1899–1929 and to 2.01% from 1929 to 1966.[28] Last, manufacturing TFP grew at an annual rate of 0.70% from 1966 to 2000.[29] Over the period 1800–2000 non-agricultural TFP grew by a factor of 9.25. Figure 9 shows the series obtained for agricultural and non-agricultural TFP.

3.2. *Parameter Values*

3.2.1. *Choice of Parameter Values*

In order to simulate the model values must be assigned to various parameters. These are listed in Table 1. Almost nothing is known about the appropriate values for some parameters, such as the lower bound on new goods consumption, $\underline{s}$, or the fixed cost associated with running a firm producing new goods, ϕ. So, the parameter values are picked to generate two steady-state equilibria that mimic some key features of the U.S. data for the years 1800 and 2000. A guide to the informal selection procedure adopted will now be given. Before proceeding, assume that a model period is 20 years and that the (annualized) rate of physical depreciation on capital is 8%.

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24 The estimates for the growth rates of agricultural productivity from 1800 to 1900 come from Atack et al. (2000, Table 6.1). (A caveat is in order since while these numbers are unambiguously reported by the authors as being TFP, they may actually represent labor productivity.)

25 The estimates for the growth in agricultural TFP for the 1900–1929 and 1929–1960 periods are computed from data in *Historical Statistics of the United States: Colonial Times to 1970* (Series W 7).

26 Source: Economic Research Service, United States Department of Agriculture, Agricultural Productivity in the U.S. (98003).

27 The estimates for technological progress in the nonagricultural sector prior to 1900 are backed out using economy-wide TFP and sectoral share data taken from Weiss (1994, Tables 1.2–1.4) and Gallman (2000, Tables 1.7 and 1.14) in conjunction with the Atack et al. (2000, Table 6.1) agricultural estimates.

28 These estimates are calculated from data in *Historical Statistics of the United States: Colonial Times to 1970* (Series W 8).

29 Source: Bureau of Labor Statistics, U.S. Department of Labor, Multifactor Productivity Trends, Table 2: Private Non-Farm Business: Productivity and Related Indexes, 1948–2001.

Table 1. Parameter values.

Tastes	$\alpha = 0.23$, $\psi = 0.22$, $\beta = 0.93^{20}$, $\sigma = 1.0–0.23–0.22$, $\underline{s} = 0.1$
Technology	$\omega = 0.46$, $\lambda = 0.11$, $\kappa = 0.28$, $\tau = 0.62$, $\phi = 0.03$, and $\delta = (1.0–0.08)^{20}$

3.2.2. U.S. Economy, ca. 1800

In 1800 agriculture accounted for 46% of U.S. output and 74% of employment. A steady state will be constructed that roughly matches these two features. To this end, normalize the initial levels of total factor productivity so that $z_a = z_c = z_i = 1$. Next, assume that no new goods were produced in 1800. This can be achieved by picking high values for $\underline{s}$ and ϕ. By doing this, output will be comprised by just agricultural and generic manufacturing goods. While it's hard to know what are reasonable values for labor's share of income in agriculture, $1-\lambda$, and in generic manufacturing, $1-\omega$, it is known that for the aggregate economy it should be about 70%. This implies that

$$\chi(1 - \lambda) + (1 - \chi)(1 - \omega) = 0.70,$$

where χ is agriculture's share of output. This restriction can be used to pin down a value for capital's share in agriculture, λ, given a value for capital's share in the generic manufacturing, ω. In other words, let

$$\lambda = 1.0 + [(1 - \chi_{1800})/\chi_{1800}](1 - \omega) - 0.70/\chi_{1800}.$$

The choice of ω, which is not observable in the data, will be discussed shortly.

3.2.3. U.S. Economy, ca. 2000

Two hundred years later agriculture's share of output and employment had dropped to just 1.4 and 2.5%, respectively. Output had increased 36.7 times.[30] Can a steady state be constructed that replicates these two facts? Over this time period total factor productivity in agriculture rose 7.61 fold. So, set $z_a = 7.61$. Similarly, total factor productivity in non-agriculture increased 9.25 times. Thus, let $z_c = z_i = 9.25$. The responsiveness of output to changes in TFP is sensitive to capital's share of income. The larger capital's share of income is the bigger will be the response. This transpires because capital is the reproducible factor in the model. The observed 36.7-fold increase in output can be obtained by setting capital's share in the generic goods sector, ω, to 0.46. New goods are produced competitively. Therefore, any profits earned in this sector are absorbed completely by the fixed costs of production. Recall that the fixed cost of producing new goods are borne entirely in terms of labor. Thus, labor's

30 This estimate is based on the data presented in Mitchell (1998, Table J1) together with the NIPA accounts.

share of income in the new goods sector is given by $(1-\kappa)$. Assuming that new goods weigh heavily in the 2000 economy, this dictates setting κ at about 30%; let $\kappa = 0.28$. To choose the exponent on labor, τ, assume that profits, and hence fixed costs, amount to 10% of new goods production so that $\tau = 1.0-\kappa-0.10 = 0.62$.

Last, three taste parameters need to be picked: α, ψ, and β. In the adopted parameterization the ca. 2000 steady state lies in Zone 2. Hence, $r = 1/\beta$.[31] An annual interest rate of about 7.5% can be achieved by setting $\beta = 0.93^{20}$. The weights on the various categories of consumption in utility are chosen to obtain the best fit matching agriculture's share of output and employment over the period 1800–2000.

4. The Gain in Welfare, 1800–2000

So by how much did welfare increase between 1800 and 2000? To address this question, define the expenditure function, $E(p_a, p'_a, \overrightarrow{p_i}, \overrightarrow{p_i}', r', u)$, by

$$E(p_a, p'_a, \overrightarrow{p_i}, \overrightarrow{p_i}', r', u) \equiv \min_{c, c^{o\prime}, a, a^{o\prime}, s_i \geq \underline{s}, s_i^{o\prime} \geq \underline{s}, I, I^{o\prime}} \left\{ c + p_a a + \frac{c^{o\prime}}{r'} + \frac{p'_a a^{o\prime}}{r'} + \int_{i=0}^{I} p_i s_i di + \int_{i=0}^{I^{o\prime}} \frac{p'_i}{r'} s_i^{o\prime} di \right\} \tag{21}$$

subject to

$$\left\{ \alpha \ln(a) + \psi \ln(c) + \sigma \int_{i=0}^{I} \ln(s_i) di + \beta[\alpha \ln(a^{o\prime}) + \psi \ln(c^{o\prime}) + \sigma \int_{i=0}^{I^{o\prime}} \ln(s_i^{o\prime}) di] + \sigma(N - I) \ln(\underline{s}) + \beta\sigma(N - I^{o\prime}) \ln(\underline{s}) \right\} = u, \tag{22}$$

where $\overrightarrow{p_i}$ represents the vector of new goods prices for the current period. The solution to this problem will be once again characterized by the first-order conditions (5)–(9), but now the choice variables must satisfy the utility constraint (22) rather than the budget constraint (4).

Consider comparing welfare across two steady states, labeled old and new. Let the subscript 0 denote a variable's value in the old steady state and the subscript T represent the variable's value in the new steady state. In the new steady state a young agent will earn w_T, face the prices $p_{a,T}$, $\overrightarrow{p_{i,T}}$, and r_T, and realize utility, u_T. In the old steady state, the young agent would have earned w_0 and realized utility u_0. Now, it would cost the amount $E(p_{a,T}, p_{a,T}, \overrightarrow{p_{i,T}}, \overrightarrow{p_{i,T}}, r_T, u_0)$ to provide the old level of utility, u_0, at the new set of prices, $p_{a,T}$, $\overrightarrow{p_{i,T}}$, and r_T. At this level of income a young agent would be indifferent between living in the new steady state or staying in the old one

31 This normally would not be the case for an overlapping generations model.

with the wage rate, w_0. Any extra income improves the agent's lot. Hence, a measure of the proportionate change in welfare across these two steady states, analogous to a compensating variation, is given by[32]

$$\ln(w_T) - \ln[E(p_{a,T}, p_{a,T}, \overrightarrow{p_{i,T}}, \overrightarrow{p_{i,T}}, r_T, u_0)].$$

Another utility-based measure is based on the concept of an equivalent variation. It measures the cost of providing the new level of utility, u_T, at the prices and set of goods that the agent faces in the old steady state, $p_{a,0}$, $\overrightarrow{p_{i,0}}$, and r_0. This gives

$$\ln[E(p_{a,0}, p_{a,0}, \overrightarrow{p_{i,0}}, \overrightarrow{p_{i,0}}, r_0, u_T)] - \ln[w_0].$$

Wages increase from w_0 to w_T across the two steady states. This does not take into account the fact that the cost of living may have also shifted due to a change in prices. The conventional way to control for this would be to deflate wages in the new steady state by a price index. The Laspeyres price index, L_T, is given by

$$L_T = \frac{(c_0 + c_0^o) + p_{a,T}(a_0 + a_0^o) + p_{i,T}(I_0 + I_0^o)e\underline{s}}{(c_0 + c_0^o) + p_{a,0}(a_0 + a_0^o) + p_{i,0}(I_0 + I_0^o)e\underline{s}}.$$

It measures the rise in the cost of purchasing the initial basket of goods. The growth in real income based on the Laspeyres price index is

$$\ln(w_T/L_T) - \ln(w_0) = \ln(w_T) - \ln(L_T w_0).$$

Of course agents would not buy the initial basket of goods in the new steady state. They would substitute toward those goods whose prices have fallen. The Paasche price index, P_T, computes the rise in the cost of living using the basket of goods consumed in the final steady state.

$$P_T = \frac{(c_T + c_T^o) + p_{a,T}(a_T + a_T^o) + p_{i,T}(I_T + I_T^o)e\underline{s}}{(c_T + c_T^o) + p_{a,0}(a_T + a_T^o) + p_{i,0}(I_T + I_T^o)e\underline{s}}.$$

The growth in real income using the Paasche price index is

$$\ln(w_T/P_T) - \ln(w_0).$$

The Fisher price index, F_T, is a geometric mean of the Laspeyres and Paasche indices so that $F_T = \sqrt{L_T P_T}$. Last, the Tornqvist index, T_T, is defined by

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32 The compensating variation, CV, associated with the move from the old to the new steady state is $E(p_{a,T}, p_{a,T}, \overrightarrow{p_{i,T}}, \overrightarrow{p_{i,T}}, r_T, u_0) - w_0$. Therefore, definitionally, $E(p_{a,T}, p_{a,T}, \overrightarrow{p_{i,T}}, \overrightarrow{p_{i,T}}, r_T, u_0) = CV + w_0$. Hence, the above welfare measure can be written as $\ln(w_T) - \ln(CV + w_0)$.

$$\ln(T_T) = (\frac{\xi_{a,0} + \xi_{a,T}}{2}) \ln(\frac{p_{a,T}}{p_{a,0}}) + (\frac{\xi_{i,0} + \xi_{i,T}}{2}) \ln(\frac{p_{i,T}}{p_{i,0}}),$$

where $\xi_{x,t}$ is the period-t expenditure share of good x (for $x = a$, i) in consumption so that, for example,

$$\xi_{a,0} = \frac{p_{a,0}(a_0 + a_0^o)}{(c_0 + c_0^o) + p_{a,0}(a_0 + a_0^o) + p_{i,0}(I_0 + I_0^o)e\underline{s}}.$$

A problem with the Paasche price index is that many of the goods purchased in the new steady state were not available in the old steady one. For example, assume that no new goods were produced in the old steady state. The price $p_{i,0}$ would not exist then. For this reason, the Laspeyres index is used in practice—the price $p_{i,0}$ would not appear in the denominator of this index since $I_0 = I_0^o = 0$ when new goods are not consumed. Hicks (1940) suggested constructing a ‘‘virtual price’’ to overcome this problem with new goods. The virtual price is the lowest price for the new good at which the consumer would choose zero units, given the prices for the other goods and his income. It is easy to construct such virtual prices in the model. To see this, assume that no new goods are consumed in the old steady state. Also suppose that $r_0 < 1/\beta$, or that the old steady state lies in Zone 1 (implying in general that $I_0 \geq 0$ and $I_0^o = 0$). Recall that if some new goods are consumed then equation (8) will hold with equality so that $s_i = e\underline{s}$. Therefore, using (6) it will transpire that

$$p_{i,0} = \frac{\sigma}{\alpha + \psi + \beta\alpha + \beta\psi + \sigma I_0} \frac{w_0}{e\underline{s}}.$$

This equation gives the inverse demand curve for new goods. To compute the virtual price, $p_{i,0}^v$, set $I_0 = 0$ in this demand relationship to obtain

$$p_{i,0}^v = \frac{\sigma}{\alpha + \psi + \beta\alpha + \beta\psi} \frac{w_0}{e\underline{s}}.$$

Some intuition for the differences between the various welfare measures is provided in Figure 10. The diagram portrays a static setting with just two types of goods, generic and new. Tastes are once again represented by (3), but now set $\alpha = \beta = 0$. Equation (10) will once again give the quantity consumed of each new good, or $s_i = e\underline{s}$. Given this, Figure 10 shows indifference curves over the quantity of generic goods, c, and the number of new goods, I, consumed. The slope of one of these indifference curves is $-\psi/(\sigma c)$. Now, imagine a situation where there are no new goods produced. Here $c = w$. This situation is portrayed by the point A. Suppose that new goods become available. Point B shows this situation. Recall that in equilibrium each new good that is produced will sell at the same price, p_i. The slope of the budget constraint is given by $-1/(p_i e\underline{s})$—the cost of consuming $e\underline{s}$ units of a new good is $p_i e\underline{s}$.[33] Clearly the consumer is better off. He is on a higher indifference curve. At the

33 The budget constraint is $c + p_i I e\underline{s} = w$.

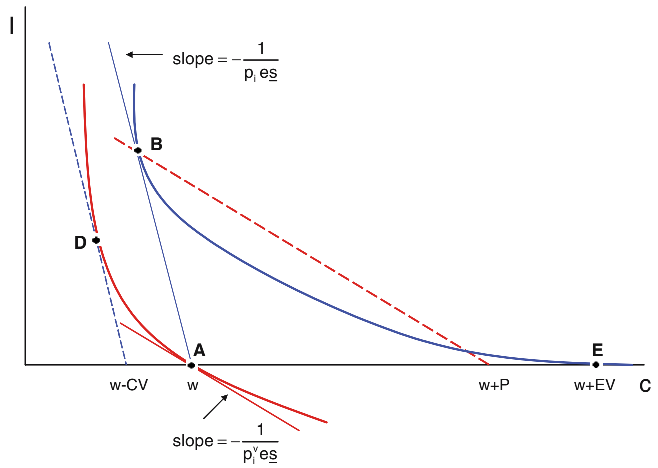

Figure 10. Welfare measures.

new prices, you could take away from the consumer CV units of income and he would remain on his old indifference curve at the point D. This shows the compensating variation. The Laspeyres price index shows no change in real income. Why? At the new set of prices the cost of the old consumption bundle is still w since *no* new goods were consumed. Hick's (1940) virtual price is given by slope of the indifference curve going through the point A. According to the Paasche index real income increases by the amount P. By giving the consumer this amount he can afford to buy the new bundle of goods, represented by point B, at the old set of (virtual) prices. Last, the distance EV measures the equivalent variation. It asks how much income would consumer have to be given in order to get his new level of utility without any new goods—see point E.

Table 2 presents the gain in welfare according to the various measures. The welfare gain due to technological progress and the introduction of new goods is large by any measure. The utility-based estimate based upon the compensating variation suggests that welfare rose by 300%, when measured in terms of generic consumption. This is a (continuously compounded) gain of about 1.5% a year. The traditional index number measures report gains very similar to the compensating variation criteria. These numbers are strikingly similar to an estimated 300% increase in the U.S. real wage over the 1800–2000 period.[34] The other utility-based estimate based upon the equivalent variation concept reports a much larger welfare gain of 2,000%. This translates into a welfare gain of about 10% a year. This measure asks by how much would income have to increase in 1800, when there were no new goods, in order to provide today's level of utility. Providing a modern utility level using just

34 This estimate is based on real wage data contained in Williamson (1995, Table A1) for the period 1830–1988. The Williamson (1995) series was updated to 2000 using data from the Bureau of Labor Statistics. The resulting series was then extrapolated back to 1800.

Table 2. Gain in welfare.

Measure	*Welfare gain, %*
Compensating variation	309
Equivalent variation	1,994
Laspeyres	280
Paasche	305
Fisher	292
Tornqvist	302

yesteryear's goods is an expensive proposition. The traditional index number concepts miss this point. Theoretically speaking, there is no good reason to prefer the compensating over the equivalent variation, or vice versa. Taking an average of the two utility-based measures suggests that welfare increased by 1,151% or grew at about 6% a year. Perhaps the safest thing to say, though, is that welfare increased by at least 300%.

5. Transitional Dynamics

5.1. The Computational Experiment

Now, imagine starting the model off in a steady state that resembles the U.S. in 1800 and letting it converge to a new steady state that resembles the U.S. in 2000. To undertake this experiment the time path for TFP shown in Figure 9 will be inputted into the simulation. The circles on the series indicate the values at 20 year intervals that will be used when simulating the model's transitional dynamics. It is simply assumed that all technological progress stops after 2000. What will the economy's behavior over this time period look like? From the earlier results it can be surmised that economy will initially be in Zone 1 and then transit into Zone 2. So before proceeding, a comment will be made about the model's local dynamics in Zone 2.[35]

5.2. Local Dynamics

The dynamics approaching the Zone 2 steady state can be characterized analytically. Recall that the price of the new good can be written as $p_i = P_i(r - \delta; z_c, z_i)$. Now, assume that the economy is in Zone 2. Equation (11) holds tightly in this Zone. It gives the following difference equation for the interest rate

$$r'\beta P_i(r - \delta; z_c, z_i) = P_i(r' - \delta; z'_c, z'_i). \tag{23}$$

To have a steady state, technological progress must eventually abate. This is assumed in Section 2. Hence, suppose that $z'_c = z_c$, and $z'_i = z_i$. What can be said about the

35 The discussion below on the model's Zone-2 local dynamics can be omitted without loss of continuity.

solution to this difference equation in this situation? The lemma below provides the answer. The cases described by the lemma are portrayed in Figure 11.

Lemma *The difference equation (23) has two rest points, viz.* $r = 1/\beta$ *and* $r = \delta$. *Its local dynamics are as follows*:

1. *When* $\kappa - \omega < 0$ *the system converges monotonically to the rest point given by* $r = 1/\beta$. *The point* $r = \delta$ *is unstable.*

2. *When* $\kappa - \omega > 0$ *two modes of behavior can happen*:

   (a) *If* $(\kappa - \omega)/(1 - \omega) < 1 - \beta\delta$ *then the system exhibits oscillations around the* $r = 1/\beta$ *rest point. These cycles converge when* $(\kappa - \omega)/(1 - \omega) < (1 - \beta\delta)/2$ *and diverge otherwise. The system converges monotonically to the point* $r = \delta$.

   (b) *Alternatively, if* $(\kappa - \omega)/(1 - \omega) > 1 - \beta\delta$ *then the rest point* $r = 1/\beta$ *is unstable. The system converges monotonically towards the point* $r = \delta$.

Proof. See Appendix.

Remark *The calibrated version of model is described by Case 1. Suppose instead that* $\kappa - \omega > 0$. *Then, as a practical matter, any reasonable calibration will result in* $(\kappa - \omega)/(1 - \omega) < (1 - \beta\delta)/2$. *This will transpire because the differences in capital shares across industries are small, and labor shares are large. Hence, for all empirically relevant equilibria the rest point* $r = 1/\beta$ *will be stable.*

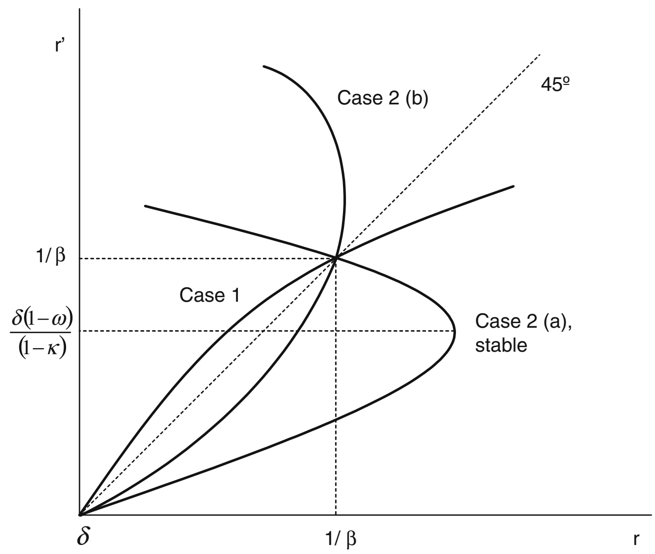

Figure 11. The model's local dynamics, Zone 2.

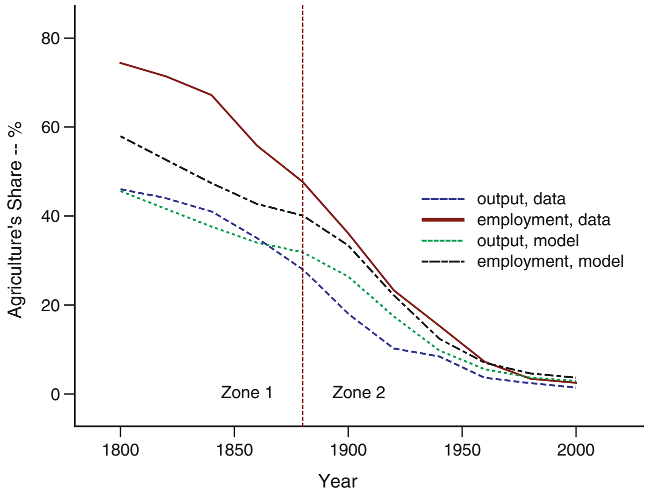

Figure 12. The decline of agriculture, 1800–2000—U.S. data and model.

5.3. Structural Change

The transitional dynamics for the model are shown in Figure 12. Given the parameterization adopted, convergence to the new steady state (where $r = 1/\beta$) is monotone. (i.e., the economy is described by Case 1 in the lemma.) The model economy transits out of Zone 1 into Zone 2 in 1880. Observe that agriculture's shares of GDP and employment decline along with technological progress.[36] The time paths predicted by the model match the data very well, with one blemish that will be discussed now.

Note that in the U.S. data, agriculture's share of employment, ϑ, significantly exceeded its share of output, χ, in 1800. This is a bit of task to achieve with a Cobb–Douglas production structure, at least when new goods are not produced. To see why, note that the efficiency conditions for employment in agriculture and generic manufacturing imply that the following relationship between relative employments and outputs must hold

$$\frac{\chi}{1-\chi} \equiv \frac{p_a y_a}{y_c} = \frac{1-\omega}{1-\lambda}\frac{l_a}{l_c} \equiv \frac{1-\omega}{1-\lambda}\frac{\vartheta}{1-\vartheta}.$$

36 The relative price of agricultural goods has an enigmatic pattern in the U.S. data. Between 1790 and 1946 it rose by a factor of roughly 1.6. It then plunged by a factor 2.3 over the interval 1946–2003. Some of this pattern could be due to index number problems associated with the introduction of new goods, as was discussed in Section 4. In addition work by Bils (2004) suggests that the failure to adjust for quality improvement in goods is a serious problem in price indices. The current analysis will sidestep this observation, given the absence of a stylized fact.

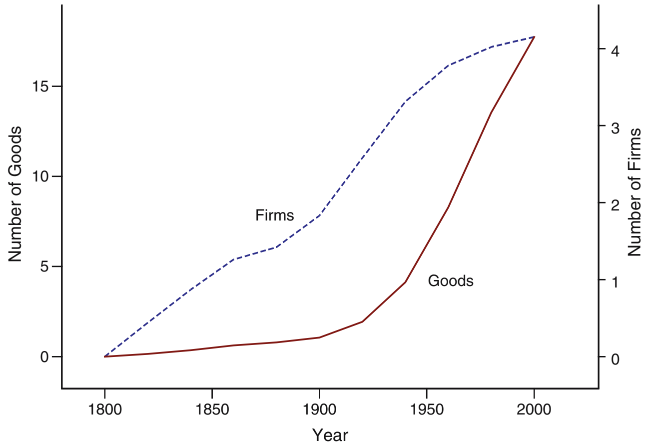

Figure 13. The rise in the number of new goods and firms, 1800–2000—model.

Hence, if agriculture is to constitute a higher fraction of employment vis à vis output then labor's share of income must be disproportionately higher in this sector; i.e., $(1-\lambda)/(1-\omega)$ must be bigger than one since $[\vartheta/(1-\vartheta)]/[\chi/(1-\chi)]$ is. This could be done by picking a high value for ω, or capital's share of income in the generic sector. The required value is 0.74. This value is unrealistic and implies that small changes in non-agricultural TFP will have enormous effects on output.

Coinciding with the fall in agriculture is the rise in new goods, as Figure 13 illustrates. As can be seen, at low levels of economic development no new goods are produced. As the state of technology progresses income rises. Workers begin to demand new goods. Both the number of new goods produced, and the number of firms producing them, rise. On some other dimensions the predictions for the model are reasonable. The interest rate is trapped between 4.6% and 8.2%. Labor's share of income hovers around 70%.

6. Conclusions

So, what is the connection between technological progress, the introduction of new goods, and the structure of production? A simple story is told here. As incomes grow, it pays for producers to introduce new goods and services. Consumers demand new goods as incomes rise because the benefit from consuming a new good is higher than the benefit from consuming more of an old good. The appealing aspect of this explanation is that in the data the importance of agriculture seems to fall unabated as economies develop, and the model developed here is consistent with that prediction. More precisely, so long as TFP increases, agriculture's share of GDP will keep on declining so long as new goods can be brought on line.

The model presented here also provides a framework, albeit crude, with which to analyze the impact that new goods have on economic welfare. The impact of technological progress on economic welfare is sizable. The exact magnitude depends on the welfare criteria used. The analysis suggests that economic welfare grew by at least 1.5% a year, and by perhaps as much as 10% a year. More elaborate versions of the model could undoubtedly do a better job. For instance, process innovation could be incorporated into the framework to capture the decline in a product's price after its introduction. At a point in time, each vintage of new goods would then be consumed in differing amounts. Over time the consumption of a new product would follow a diffusion curve. This may create more powerful substitution effects that could cause more divergence among the various indices of welfare. A model provides an ideal laboratory to evaluate the performance of the various indices.

7. Appendix

7.1. The Lemma

By using the first-order conditions to problem (14), it can be deduced that the profits earned by a firm in the new goods sector will be given by

$$\begin{aligned}\pi_i &= (1-\kappa-\tau)\kappa^{\kappa/(1-\kappa-\tau)}\tau^{\tau/(1-\kappa-\tau)}(p_i z_i)^{1/(1-\kappa-\tau)}\\ &\quad\times(r-\delta)^{-\kappa/(1-\kappa-\tau)}w^{-\tau/(1-\kappa-\tau)} - w\phi = 0.\end{aligned}$$

Next, the first-order conditions to problem (12) imply that

$$w = W(r-\delta; z_c) \equiv (1-\omega)z_c\left(\frac{r-\delta}{\omega z_c}\right)^{\omega/(\omega-1)}. \tag{A.1}$$

Using the above two equations allows the price for new goods to be written as

$$\begin{aligned}p_i = P_i(r-\delta; z_c, z_i) &\equiv z_i^{-1}\left[\frac{\phi}{(1-\kappa-\tau)}\right]^{(1-\kappa-\tau)}\kappa^{-\kappa}\tau^{-\tau}\\ &\quad\times(1-\omega)^{1-\kappa}\omega^{(1-\kappa)\omega/(1-\omega)}z_c^{(1-\kappa)/(1-\omega)}(r-\delta)^{[\kappa-\omega]/(1-\omega)}.\end{aligned} \tag{A.2}$$

It is then straightforward to calculate that

$$P_{i1}(r-\delta; z_c, z_i) = \frac{1}{r-\delta}\frac{\kappa-\omega}{1-\omega}P_i(r-\delta; z_c, z_i). \tag{A.3}$$

With (A.2) in hand, it is easy to see that the difference equation (23) can be rewritten as

$$r'\beta(r-\delta)^{[\kappa-\omega]/(1-\omega)} = (r'-\delta)^{[\kappa-\omega]/(1-\omega)}, \tag{A.4}$$

when $z_c' = z_c$ and $z_i' = z_i$. What can be said about the solutions to this equation?

Lemma *The difference equation (23) has two rest points, viz.* $r = 1/\beta$ *and* $r = \delta$. *Its local dynamics are as follows*:

1. *When* $\kappa - \omega < 0$ *the system converges monotonically to the rest point given by* $r = 1/\beta$. *The point* $r = \delta$ *is unstable.*

2. *When* $\kappa - \omega > 0$ *two modes of behavior can happen*:

   (a) *If* $(\kappa - \omega)/(1 - \omega) < 1 - \beta\delta$ *then the system exhibits oscillations around the* $r = 1/\beta$ *rest point. These cycles converge when* $(\kappa - \omega)/(1 - \omega) < (1 - \beta\delta)/2$ *and diverge otherwise. The system converges monotonically to the point* $r = \delta$.

   (b) *Alternatively, if* $(\kappa - \omega)/(1 - \omega) > 1 - \beta\delta$ *then the rest point* $r = 1/\beta$ *is unstable. The system converges monotonically towards the point* $r = \delta$.

Proof. Rewrite the mapping given by (23) as

$$r' = D(r; z_c, z_i). \tag{A.5}$$

(Recall that in a steady state, $z_c' = z_c$ and $z_i' = z_i$.) It is clear from (A.4) that $r' = r = 1/\beta$ and $r' = r = \delta$ are both rest points to this equation. By the implicit function theorem it can be seen from (23) that D is a C^1 function with

$$\frac{dr'}{dr} = D_1(r; z_c, z_i) = \frac{r'\beta P_{i1}(r - \delta; z_c, z_i)}{P_{i1}(r' - \delta; z_c, z_i) - \beta P_i(r - \delta; z_c, z_i)}. \tag{A.6}$$

Using (23), (A.2), and (A.3) in (A.6) calculate that

$$\begin{aligned}\frac{dr'}{dr} = D_1(r; z_c, z_i) &= \frac{[(\kappa - \omega)/(1 - \omega)]r'(r'\beta)^{(1-\omega)/(\kappa-\omega)}}{[(\kappa - \omega)/(1 - \omega)]r' - (r' - \delta)} \\ &= \frac{[(\kappa - \omega)/(1 - \omega)]r'(r'\beta)^{(1-\omega)/(\kappa-\omega)}}{\Delta(r')}.\end{aligned} \tag{A.7}$$

Case 1. First, if $\kappa - \omega < 0$ then $P_{i1}(r - \delta; z_c, z_i)$ and $P_{i1}(r' - \delta; z_c, z_i) < 0$. From (A.6) it is easy to see that $dr'/dr = D_1(r; z_c, z_i) > 0$ for all r and r' combinations. Therefore, the law of motion D rises continuously from the point $r' = r = \delta$, and converges asymptotically to $\lim_{r\to\infty} D(r; z_c, z_i) = \infty$ [as is evident from (A.4)]. It is also easy to deduce that $dr'/dr|_{r'=r=1/\beta} = D_1(1/\beta; z_c, z_i) < 1$; in fact, $D_1(r; z_c, z_i) < 1$ whenever $r > 1/\beta$. Furthermore, observe from (A.7) that $dr'/dr|_{r'=r=\delta} = D_1(\delta; z_c, z_i) = (\delta\beta)^{(1-\omega)/(\kappa-\omega)} > 1$. Hence, the system converges monotonically to the rest point $r' = r = 1/\beta$ from any $r \neq \delta$.

Case 2. Second, suppose that $\kappa - \omega > 0$. From (A.7) it is apparent that

$$\frac{dr'}{dr} \gtreqless 0 \text{ as } \Delta(r') \gtreqless 0.$$

In turn it is easy to compute that

$$\Delta(r') \gtreqqless 0 \text{ as } r' \lesseqqgtr \delta(\frac{1-\omega}{1-\kappa}).$$

[Note that $(1-\omega)/(1-\kappa) > 1$ when $\kappa - \omega > 0$.] Consequently, the law of motion D approaches the point $r' = \delta(\frac{1-\omega}{1-\kappa}) \equiv \xi$ and $r = (\xi - \delta)/(\xi\beta)^{(1-\omega)/(\kappa-\omega)} + \delta$ from two ways: (i) upwards from below, and (ii) downwards from above. That is, it starts off from $r' = r = \delta$ and rises upwards to $r' = \delta(\frac{1-\omega}{1-\kappa}) \equiv \xi$ and $r = (\xi - \delta)/(\xi\beta)^{(1-\omega)/(\kappa-\omega)} + \delta$. It then bends backwards, and as r returns to δ, the law of motion D asymptotes to $\lim_{r\to\delta} D(r; z_c, z_i) = \infty$ [as is again evident from (A.4)]. A portrayal of this situation is given in Figure 11 for Case 2(a). Now, from (A.7) it is obvious that $0 < dr'/dr|_{r'=r=\delta} = D_1(\delta; z_c, z_i) = (\delta\beta)^{(1-\omega)/(\kappa-\omega)} < 1$. Therefore, the point $r' = r = \delta$ is locally stable.

What about the other rest point, $r' = r = 1/\beta$? Two subcases occur depending on whether $\Delta(1/\beta) \gtreqqless 0$. To begin with, note that

$$\Delta(1/\beta) \gtreqqless 0 \text{ as } \frac{\kappa-\omega}{1-\omega} \gtreqqless 1 - \beta\delta.$$

Now assume that $\Delta(1/\beta) > 0$. It follows from (A.7) that $dr'/dr|_{r'=r=1/\beta} > 1$. In this case the rest point $r' = r = 1/\beta$ is unstable. Alternatively, suppose that $\Delta(1/\beta) < 0$. Here, the system oscillates around the rest point $r' = r = 1/\beta$. Are these oscillations locally stable? Equation (A.7) implies that

$$dr'/dr|_{r'=r=1/\beta} \gtreqqless -1 \text{ as } \frac{\kappa-\omega}{1-\omega} \lesseqqgtr (1-\beta\delta)/2.$$

The stable solution is shown in Figure 11 for Case 2(a).

7.2. Transitional Dynamics Solution Algorithm

Pick a T large enough so that convergence takes place within $T+1$ periods, so that all variables in the model will take their steady-state values by period $T+1$. Start iteration j with a guess for the interest rate path, $\{r_t\}_{t=0}^T$, and the time path for the number of new goods consumed by the young, $\{I_t\}_{t=0}^T$, denoted by $\{r_t^j\}_{t=0}^T$ and $\{I_t^j\}_{t=0}^T$, respectively. Now, with a little bit of work, it can be shown that

$$p_a = P_a(r-\delta; z_a, z_c) \equiv \frac{(r-\delta)^{\lambda} w^{1-\lambda}}{z_a \lambda^{\lambda}(1-\lambda)^{(1-\lambda)}}, \tag{A.8}$$

where w is given by (A.1). Hence, a guess can be obtained, using (A.8), (A.2) and (A.1), for the price and wage paths $\{p_{a,t}\}_{t=0}^T$,$\{p_{i,t}\}_{t=0}^T$, and $\{w_t\}_{t=0}^T$. Represent this by $\{p_{a,t}^j\}_{t=0}^T$,$\{p_{i,t}^j\}_{t=0}^T$, and $\{w_t^j\}_{t=0}^T$.

Time period t: In time period t the state variables will be $\mathbf{k}_t$ and r_t. Given the guess $\{r^j_{x+1}\}^T_{x=t+1}$ and $\{I^j_x\}^T_{x=t+1}$, a solution for either r_{t+1} or I_t must be found, depending on whether the model is in Zone 1 or Zone 2. This is done using the capital market-clearing condition.

$$\mathbf{k}_{t+1} = b_{t+1}.$$

The supply of capital, b_{t+1}, derives from the optimization problem (3) for the period-t young. It is equal to their savings so that

$$b_{t+1} = w_t - c_t - p_{a,t}a_t - I_t p_{i,t} e\underline{s}.$$

The demand for capital, $\mathbf{k}_{t+1}$, reads

$$\mathbf{k}_{t+1} = k_{a,t+1} + k_{c,t+1} + \max\{I_{t+1}, I^o_{t+1}\} n_{i,t+1} k_{i,t+1}.$$

In the above equation the superscript o denotes an allocation by an old agent. Furthermore I_{t+1} is determined by the time-$(t+1)$ solution to (3), while I^o_{t+1} is determined by the time$-t$ solution to (3)—the solutions will depend upon what zone the model is in. The period-$(t+1)$ number of firms in the new goods sector i, $n_{i,t+1}$, is given by

$$n_{i,t+1} = \left[\frac{(I_{t+1} + I^0_{t+1})/\max\{I_{t+1}, I^0_{t+1}\}}{z_{i,t+1} k^{\kappa}_{i,t+1} l^{\tau}_{i,t+1}}\right] e\underline{s}.$$

The demand for capital in the agricultural sector is

$$k_{a,t+1} = \frac{\mathbf{a}_{t+1}}{z_{a,t+1}(k_{a,t+1}/l_{a,t+1})^{\lambda-1}},$$

where

$$\mathbf{a}_{t+1} = a_{t+1} + a^o_{t+1}.$$

Note that a_{t+1} will be determined by the time-$(t+1)$ solution to (3) while a^o_{t+1} will obtain from the time-t solution to this problem. In the model all period-$(t+1)$ capital-labor ratios, such as $k_{a,t+1}/l_{a,t+1}$, can be expressed as functions of the period-$(t+1)$ interest rate, r_{t+1}—recall that w_{t+1} is a function of r_{t+1}. In a similar vein the capital stock employed in the new goods sector is

$$k_{i,t+1} = \left[\frac{\mathbf{s}_{i,t+1}}{z_{i,t+1}(k_{i,t+1}/l_{i,t+1})^{-\tau}}\right]^{1/(\kappa+\tau)},$$

where

$$\mathbf{s}_{i,t+1} = \frac{I_{t+1}}{\max\{I_{t+1}, I^o_{t+1}\}} e\underline{s} + \frac{I^o_{t+1}}{\max\{I_{t+1}, I^o_{t+1}\}} e\underline{s}.$$

Again, note that $k_{i,t+1}/l_{i,t+1}$ can be written as a function of r_{t+1}.

The period-$(t+1)$ market-clearing condition for generic manufacturing goods is

$$\mathbf{c}_{t+1} + \mathbf{k}_{t+2} - \delta\mathbf{k}_{t+1} = z_{c,t+1}k_{c,t+1}(k_{c,t+1}/l_{c,t+1})^{\omega-1},$$

which implies that

$$\begin{aligned} k_{c,t+1} = &\{\mathbf{c}_{t+1} + \mathbf{k}_{t+2} - \delta[k_{a,t+1} + n_{i,t+1}\max\{I_{t+1}, I^o_{t+1}\}k_{i,t+1}]\} \\ &/[z_{c,t+1}(k_{c,t+1}/l_{c,t+1})^{\omega-1} + \delta]. \end{aligned}$$

Here aggregate generic manufacturing consumption, $\mathbf{c}_{t+1}$, is

$$\mathbf{c}_{t+1} = c_{t+1} + c^o_{t+1},$$

where c_{t+1} and c^o_{t+1} are given by the time-$(t+1)$ and time-t solutions to (3). Note that $\mathbf{k}_{t+2}$ can readily be computed from time-$(t+1)$ aggregate savings.

By tracing through the above equations, it can be seen that, given a guess for $\{r^j_{x+1}\}^T_{x=t+1}$ and $\{I^j_x\}^T_{x=t+1}$, everything can be solved out for in terms of just either r_{t+1} or I_t depending upon whether the model is in Zone 1 or Zone 2. When the model is in Zone 2 then r_{t+1} is pinned down by the difference equation $p_{i,t+1}/r_{t+1} = \beta p_{i,t}$. [Note that the period-$t$ young agent's intertemporal budget constraint (4) implies that solving out for I_t is the same thing as solving out for I^o_{t+1}.[37] The variable I_{t+1} comes from the guess path.] When the model is in Zone 1 then I_t (or equivalently I^o_{t+1}) is determined by the solution to the optimization problem (3) as a function of r_{t+1}.[38]

Initial Period 0: At time zero there is an unanticipated wealth redistribution given the unexpected shift in technology. Hence, the initial interest rate, r_0, that clears the capital market must also be computed. There are now two variables that need to be solved for: r_0, and either r_1 or I_0. That is, the initial interest rate is not a state variable that has been determined in the previous period. The solution for either r_1 or I_0 obtains in the manner described above. The solution for r_0 is achieved by adding the time-0 capital market-clearing condition

37 It is easy to calculate that in Zone 2

$$I^o_{t+1} = w_t/(\beta p_{i,t}e\underline{s}) - (\alpha + \psi + \beta\alpha + \beta\psi)/(\sigma\beta) - I_t/\beta.$$

38 In line with (11), when $p_{i,t} < p_{i,t+1}/(r_{t+1}\beta)$ it transpires that $I^o_{t+1} = 0$. In Zone 3 (which never occurs in the computational work) $p_{i,t} > p_{i,t+1}/(r_{t+1}\beta)$. Here,

$$I^o_{t+1} = w_t r_{t+1}/(p_{i,t+1}e\underline{s}) - (\alpha + \psi + \beta\alpha + \beta\psi)/(\beta\sigma).$$

$$k_{a,0} + k_{c,0} + \max\{I_0, I_0^o\} n_{i,0} k_{i,0} = \mathbf{k}_0.$$

The demand for capital in the agricultural sector is given by

$$k_{a,0} = \frac{\mathbf{a}_0}{z_{a,0}(k_{a,0}/l_{a,0})^{\lambda-1}},$$

where

$$\mathbf{a}_0 = a_0 + a_0^o.$$

Here the solution for a_0^o obtains from

$$a_0^o = \frac{\alpha}{\alpha + \psi + \sigma I_0^o} r_0 \mathbf{k}_0 / p_{a,0}.$$

In a similar vein the capital stock employed in the new goods sector is

$$\max\{I_0, I_0^o\} n_{i,0} k_{i,0} = \max\{I_0, I_0^o\} n_{i,0} \left(\frac{(r_0 - \delta)}{\kappa p_{i,0} z_{i,0} (k_{i,0}/l_{i,0})^{-\tau}} \right)^{1/(\kappa+\tau-1)}.$$

In the above equation I_0 is determined by the time-0 solution to (3) while I_0^o will be specified by

$$I_0^o = \max\{\frac{r_0 \mathbf{k}_0}{p_{i,0} e \underline{s}} - \frac{\alpha + \psi}{\sigma}, 0\}.$$

The market-clearing condition for generic manufacturing goods is

$$\mathbf{c}_0 + \mathbf{k}_1 - \delta \mathbf{k}_0 = z_{c,0} k_{c,0} (k_{c,0}/l_{c,0})^{\omega-1},$$

which implies that

$$k_{c,0} = \{\mathbf{c}_0 + \mathbf{k}_1 - \delta \mathbf{k}_0\} / [z_{c,0}(k_{c,0}/l_{c,0})^{\omega-1}].$$

Here aggregate manufacturing consumption, $\mathbf{c}_0$, is given by

$$\mathbf{c}_0 = c_0 + c_0^o,$$

where c_0 derives from (3) while c_0^o is determined by

$$c_0^o = \frac{\psi}{\alpha + \psi + \sigma I_0^o} r_0 \mathbf{k}_0.$$

The algorithm: The algorithm proceeds by iterating down the time path starting at time 0 and moving on to time period T. The solution $\{r_t, I_t\}_{t=0}^{T}$ obtained at each iteration j is used as a revised guess for iteration $j+1$. The algorithm continues until $\{r_t^j, I_t^j\}_{t=0}^{T} \rightarrow \{r_t^{j+1}, I_t^{j+1}\}_{t=0}^{T}$.

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